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Poincaré duality and Serre fibrations [☆]

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Abstract

We prove that a proper map $f : M^m \to N^n$ between manifolds is a Serre fibration if it has the homotopy lifting property for $(m-n)$-dimensional polyhedra, where n is close to $m/2$. © 1997 Elsevier Science B.V.

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0. Introduction

To detect a homotopical sphere it is sufficient to prove connectedness of the considered manifold up to the middle dimension. In this paper we demonstrate the analog of this phenomenon in the detection of Serre fibrations of manifolds. Our main theorem states.

Theorem 1. *Let $f : M^m \to N^n$ be a proper Serre k-fibration of manifolds without boundary such that $m - n \leqslant k$. Then f is a Serre fibration if $m = 2k + 1$ or $m = 2k + 2$ and f has no singular points.*

Definition 2. A continuous mapping satisfying the covering homotopy axiom for polyhedra of dimension at most k is said to be a *Serre k-fibration*, or *k-fibration*.

Definition 3. If a point $x \in X$ is isolated in the fibre $f^{-1}(f(x))$, then the point x is called *singular*.

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The following example shows the sharpeness of the conditions of Theorem 1. Consider the Hopf fibration $h: S^3 \to S^2$. Then suspension Σh maps S^4 to S^3 and is not a 2-fibration. That Σh is a 1-fibration follows from [7]. Clearly, this suspension has two singular points.

For manifolds with boundary, one has an analogous theorem with more strict restrictions on dimension.

Theorem 4. *Let $f: M^m \to N^n$ be a proper Serre k-fibration of manifolds with boundary such that $m - n < k$. Then f is a Serre fibration if $m \leqslant 2k + 2$.*

The proof of the above theorems are based on the following more exact theorem announced in [7].

Theorem 5. *If $f: M^m \to N^n$ is a homotopically r-regular $(r \geqslant 0)$ mapping of manifolds, then f is infinitely regular at a point $x \in \operatorname{Int} M$, if one of the following conditions is satisfied:*

(1) *$m \leqslant 2r + 2$ and $m - n \leqslant r$,*
(2) *$m = 2r + 3$, $n > r + 2$ and f is an approximate $(r + 1)$-fibration,*
(3) *$m = 2r + 3$, $n = r + 2$, $f(x) \in \operatorname{Int} N$ and f is an approximate $(r + 1)$-fibration,*
(4) *$m = 2r + 4$, $n > r + 3$ and f is an $(r + 1)$-fibration,*
(5) *$m = 2r+4$, $n = r+3$, x is not singular, $f(x) \in \operatorname{Int} N$ and f is an $(r+1)$-fibration.*

All spaces considered are separable metric, all mappings are continuous.

Definition 6. A *lift* of a map $g: Z \to Y$ relative to $f: X \to Y$ is a map $\widetilde{g}: Z \to X$ such that $f \circ \widetilde{g} = g$. A map $f: X \to Y$ is (*locally*) *soft* relative to a pair (A, B) (B is a space, and A is its subspace) if for any map $\varphi: B \to Y$ and its partial lift $\psi: A \to X$ there exists a lift $\widetilde{\varphi}$ (over some neighborhood of A) which extends ψ. A map is called (*locally*) *n-soft* if it is (locally) soft relative to every pair (A, B) with B a paracompact space of dimension $\dim B \leqslant n$ and A a closed subspace of B.

By I we denote the unit interval $[0, 1]$, while S^k and B^k denote the k-dimensional sphere and closed ball respectively. The Hilbert cube $\prod_1^\infty [0, 1]$ with usual metric is denoted by I^∞.

Definition 7. A map $f: X \to Y$ is said to be *approximately* (A, B)*-soft* if for every mapping $\varphi: B \to Y$ and for each of its partial lifts $\psi: A \to X$ there exists a sequence of maps $\widetilde{\varphi}_i: B \to X$ such that $\widetilde{\varphi}_i|_A = \psi$ and the sequence $f \circ \widetilde{\varphi}_i$ uniformly converges to φ. A map f is called an *approximate n-fibration* if it is approximately soft relative to every pair $(P \times I, P \times 0)$, where P is an n-polyhedron.

We shall consider the singular homology (denoted by H_k) and Čech homology with compact supports (denoted by $\check{H}_k$). Let $T_{s,c}: H_k \to \check{H}_k$ be a natural transformation between two homology theories. Coefficients will be usually the integers $\mathbb{Z}$, in which case they will be omitted from notation.

Definition 8. A mapping $f : X \to Y$ is said to be *homotopically* (respectively *homologically*) *n-regular* at a point $x \in X$ if for every open $U \ni x$ there exist open $V \ni x$ and open $W \ni f(x)$ such that every mapping $g : S^k \to V \cap f^{-1}(y)$ (respectively every cycle $z \in \check{H}_k(V \cap f^{-1}(y))$), where $y \in W$ and $0 \leqslant k \leqslant n$, is homotopic to the constant map (respectively homologous to zero) in $U \cap f^{-1}(y)$. For the brevity we call a homotopically n-regular map h-n-regular and a homologically n-regular map H-n-regular.

Now homotopically (respectively homologically) n-regular mapping can be defined as a mapping which is homotopically (respectively homologically) n-regular at every point of its domain.

It follows from the finite-dimensional selection theorem of Michael [3] that a mapping of finite-dimensional manifolds is h-n-regular if and only if it is locally $(n+1)$-soft.

1. Proofs of the theorems

The arguments of Theorem 2.4 from [2] allow us to prove the following lemma:

Lemma 9. *Suppose that $f : M \to N$ is a homotopically r-regular mapping of finite-dimensional manifolds and* $\dim f^{-1}(y) \leqslant r$ *for every point $y \in N$. Then f is homotopically infinitely regular.*

Definition 10. A subset A of metric space X is said to be *k-negligible* if for every mapping $f : I^k \to X$ there exists a mapping $f' : I^k \to X$ which is arbitrarily close to f and such that $f(I^k) \cap f'(I^k) = \emptyset$.

Since the mapping f is h-r-regular and every point $y \in N$ is $(n-1)$-negligible in the manifold N, every fibre $f^{-1}(y)$ is $\min\{n-1, r+1\}$-negligible in the manifold M [6]. Therefore $\dim f^{-1}(y) \leqslant m - 1 - \min\{n-1, r+1\}$.

Proof of Theorem 5(1). It follows from inequalities (1) that $\dim f^{-1}(y) \leqslant r$ for every point $y \in Y$. Then by Lemma 1 the mapping f is h-∞-regular at every point $x \in \operatorname{Int} M$. $\square$

Definition 11. For a compact K a cycle $z \in \check{H}_n(K)$ is called *inessential* if z is homologous to zero on each of its support. In the converse case a cycle z is called *essential.*

A spheroid is called *homotopically inessential* if it is contractible in its image.

Proposition 12. *Suppose that the assumptions of Theorem 5 and one of the conditions* (2)–(5) *are satisfied. Then there exists an open neighborhood U of the point x such that every cycle $z \in \check{H}_{r+1}(U \cap f^{-1}(y))$ is inessential.*

Proof of Theorem 5(2)–(5). Let us deduce our Theorem 5 from Proposition 12.

If $r = 0$, then every fibre $f^{-1}(y)$ is locally connected. By Proposition 12 every cycle $z \in H_1(U \cap f^{-1}(y))$ is inessential. Therefore every mapping $g: S^1 \to U \cap f^{-1}(y)$ is homotopically inessential (i.e., is contractible in the range $g(S^1)$). Hence $f|_U$ is homotopically 1-regular.

If $r \geqslant 1$ then by Proposition 12 the mapping $f|_U$ is homologically $(r+1)$-regular. Using arguments of Lemma 2.7 from [2] one can prove that a mapping $f: M \to N$ of finite-dimensional manifolds is h-$(r+1)$-regular over an open set $U \subset M$ if $f|_U$ is h-r-regular and H-$(r+1)$-regular.

Since $f|_U$ is h-$(r+1)$-regular, the set $f^{-1}(y) \cap U$ is $\min\{n-1, r+2\}$-negligible in the manifold M [6, Corollary 2]. We have $\min\{n-1, r+2\} \geqslant r+1$ in cases (2) and (3), and $\min\{n-1, r+2\} = r+2$ in cases (4) and (5). Therefore $\dim(f^{-1}(y) \cap U) \leqslant m - 1 - \min\{n-1, r+2\} = r+1$.

Consequently the mapping $f|_U: U \to N$ is h-$(r+1)$-regular and dimension of its fibers is less than or equal to $r+1$. By Lemma 9 the mapping $f|_U$ is homotopically ∞-regular. $\square$

Proof of Proposition 12. Let us prove Proposition 12 independently for the cases (2)–(5). In every case we find a neighborhood U of the point x such that $U \cong \mathbb{R}^m$ and $f(\overline{U}) \neq N$.

Assuming the converse we construct a rational cycle z, which is not homologous to zero on its support $K \subset U$, and then we construct a spheroid $\varphi: S^{r+1} \to M \setminus f^{-1}(f(K))$ such that φ is linked with z whereas the spheroid $f \circ \varphi: S^{r+1} \to N$ is contractible missing $f(K)$.

Fix a point $q \in N \setminus f(\overline{U})$. Let $h_t: S^{r+1} \to N \setminus f(K)$ be a homotopy that contracts the map $f \circ \varphi$ to the point q. Let $\widetilde{h}_t: S^{r+1} \to M$ be a lift (approximate lift in cases (2) and (3)) of the homotopy h_t such that $\widetilde{h}_0 = \varphi$, the homotopy $f \circ \widetilde{h}_t$ avoid $f(K)$, and $f \circ \widetilde{h}_1: S^{r+1} \to N \setminus f(\overline{U})$. Then the homotopy $\widetilde{h}$ avoid the set K and $\widetilde{h}_1(S^{r+1}) \cap \overline{U} = \emptyset$.

We have $\langle \widetilde{h}_{1*}(S^{r+1}), z\rangle = \langle \varphi_*(S^{r+1}), z\rangle \neq 0$. Since $\widetilde{h}_1(S^{r+1}) \cap \overline{U} = \emptyset$ and the cycle z is homologous to zero in $U \cong \mathbb{R}^m$, then $\langle \widetilde{h}_{1*}(S^{r+1}), z\rangle = 0$. This contradiction proves Proposition 12.

It remains to construct a neighborhood U of the point x, a cycle z, and a spheroid φ in each of the cases (2)–(5).

Case (2). Let U be an open neighborhood of the point x such that $U \cong \mathbb{R}^m$ and $f(\overline{U}) \neq N$.

Suppose that there exist a point $y \in N$ and essential cycle $z' \in \check{H}_{r+1}(U \cap f^{-1}(y))$ which is not homologous to zero on its support K. Since $\dim f^{-1}(y) \leqslant r+1$, then z' is of infinite order. Then the coefficient homomorphism takes z' to a cycle

$$z \in \check{H}_{r+1}(U \cap f^{-1}(y); \mathbb{Q}).$$

There is a cycle $v \in H_{r+1}(U \setminus K)$ which is linked with z [1, Chapter 3]. Since the fibre $f^{-1}(y)$ is $(r+1)$-negligible, we may assume that $v \in H_{r+1}(U \setminus f^{-1}(y))$ and that there exists an $(r+2)$-chain $u \subset W$ such that $\Delta u = v$ and the $(r+1)$-skeleton $u^{(r+1)}$

does not intersect $f^{-1}(y)$. Then there is a simplex $\sigma \in u$ such that $\Delta\sigma$ is linked with z. Take a mapping $\varphi: S^{r+1} \to U \setminus f^{-1}(y)$ such that $\varphi_*(S^{r+1}) = \Delta\sigma$.

Clearly, φ is linked with z. Since $\dim N > r+3$, the spheroid $f \circ \varphi: S^{r+1} \to N$ is contractible missing the point $y = f(K)$. So, the neighborhood U, the cycle z, and the spheroid φ are as desired.

Case (3). Let us show that the point x is not singular. Take an open neighborhood $V \cong \mathbb{R}^{r+2}$ of the point $y' = f(x)$. Take a sequence of maps $\{g_i: S_i^{r+1} \to V \setminus \{y'\}\}_{i=1}^{\infty}$ such that $\lim_{i\to\infty} g_i(S_i^{r+1}) = \{y'\}$ and g_i is not homotopic to a constant map in $V \setminus \{y'\}$. Since f is locally $(r+1)$-soft, there exists a sequence of lifts $\{\widetilde{g}_i: S_i^{r+1} \to M\}_{i\geqslant K}$ such that $\lim_{i\to\infty} \widetilde{g}_i(S_i^{r+1}) = \{x\}$. Note that $\widetilde{g}_i$ is not homotopic to a constant map in $f^{-1}(V) \setminus f^{-1}(y')$. Therefore the component of the point x in the fibre $f^{-1}(y')$ is nontrivial. Fix a point $x' \neq x$ in this component.

Since f is homotopically 0-regular, there exist open neighborhoods Ox and Ox' of the points x and x' such that any pair of points $p \in Ox$, $p' \in Ox'$ lying in some fibre of the map f may be connected by a path in this fibre. Take the neighborhoods $U \subset Ox$ and $U' \subset Ox'$ of the points x and x', which are homeomorphic to $\mathbb{R}^m$, and such that $f(\overline{U}) \subset f(U') \neq N$.

Suppose that there exist a point $y \in N$ and an essential cycle $z' \in \check{H}_{r+1}(U \cap f^{-1}(y))$ which is not homologous to zero on its support K. Arguing as in Case (2), we obtain a cycle $z \in \check{H}_{r+1}(U \cap f^{-1}(y); \mathbb{Q})$, linked with a cycle $v \in H_{r+1}(U \setminus K)$, and an $(r+2)$-chain $u \subset U$ such that $\Delta u = v$ and the $(r+1)$-skeleton $u^{(r+1)}$ does not intersect $f^{-1}(y)$. Taking small triangulations of the chain u, we can find a sequence of simplexes $\{\sigma_i\}_{i=1}^{\infty} \subset U$ that converges to the point $x_0 \in U \cap f^{-1}(y)$ and such that the boundaries $\Delta\sigma_i \in u^{(r+1)}$ are linked with z. Fix a point $s \in S^{r+1}$. Consider the mappings $\varphi_i: S^{r+1} \to U \setminus f^{-1}(y)$ such that $\varphi_*^i(S^{r+1}) = \Delta\sigma_i$ and the sequence $\{\varphi^i(S^{r+1})\}$ converges to the point x_0. Then $f \circ \varphi^i(S^{r+1}) \to_{i\to\infty} y$. Take a point $x_0' \in U' \cap f^{-1}(y)$. Since f is locally $(r+1)$-soft, then for sufficiently large i there exist mappings $\psi^i: S^{r+1} \to M$ such that $\lim_{i\to\infty} \psi^i(S^{r+1}) = x_0'$ and $f \circ \psi^i = f \circ \varphi^i$. Fix a number j such that $\psi^j(S^{r+1}) \subset U'$. Clearly, the cycle $\psi_*^j(S^{r+1})$ does not link with z.

Consider a mapping η of the unit sphere $S_0^{r+1} \subset \mathbb{R}^{n+2}$ that takes every parallel, having $(n+2)$th coordinate less than or equal to $\frac{1}{3}$, to a point. Then $\eta(S_0^{r+1})$ is homeomorphic to $S_1^{r+1} \cup J \cup S_2^{r+1}$, where J is an arc which intersect spheres S_1^{r+1} and S_2^{r+1} in the points s_1 and s_2 respectively. Consider a mapping $\xi: \eta(S_0^{r+1}) \to M$ such that

(1) $\xi|_{S_1^{r+1}} = \psi^i \circ \alpha$, where $\alpha: (S_1^{r+1}, s_1) \to (S^{r+1}, s)$ is a homeomorphism,

(2) the path $\xi(J)$ lies in a fibre of the map f,

(3) $\xi|_{S_2^{r+1}} = \varphi^j \circ \beta$, where the homeomorphism $\beta: (S_2^{r+1}, s_2) \to (S^{r+1}, s)$ is chosen so that the map $f \circ \xi \circ \eta: S_0^{r+1} \to N$ is homotopically inessential.

Let us put $\varphi = \xi \circ \eta$. Clearly, the cycle $\varphi_* \in H_{r+1}(M \setminus f^{-1}(y))$ is linked with z.

Case (4). Let U and W be open neighborhoods of the point x such that $f(W) \neq N$, $\overline{U} \subset W$, and $U \cong \mathbb{R}^m$. Suppose that there exists a point $y \in N$ and an essential cycle $z'' \in \check{H}_{r+1}(U \cap f^{-1}(y))$ not homological to zero on its support K'. Then the cycle z'' is not homologous to zero also in some neighborhood OK' of the compact K' in the fibre $f^{-1}(y) \cap U$. As the fibre $f^{-1}(y)$ is locally connected up to dimension r, there exists a finite polyhedron P of the dimension $r+1$, a map $g: P \to OK'$ and a cycle $z' \in H_{r+1}(P)$ such that $T_{s,c} \circ g_*(z') = z''$.

Fix a sequence of distinct points $\{y_i\}_{i=1}^{\infty}$ in $\operatorname{Int} N \setminus \{y\}$ that converges to the point y. Since $n > 2$, one can define a mapping $\theta: I \times I \to N$ such that $\theta(I \times \{0\}) = y$, $\theta(I \times \{1/i\}) = y_i$ for any $i \in \mathbb{N}$, and $\theta(\lambda, t) = \theta(\lambda', t')$ iff $t = t' = 0$ or $t = t' = 1/i$ for some $i \in \mathbb{N}$. Since f is an $(r+1)$-fibration, then for any $\lambda \in I$ there exists a homotopy $\Theta^\lambda: P \times I \to M$ of the map $g: P \to M$ such that $f \circ \Theta^\lambda(P \times \{t\}) = \theta(\lambda, t)$. For every $\lambda \in I$ take a number $l(\lambda) \in \mathbb{N}$ so that $\Theta^\lambda(P \times [0, 1/l(\lambda)]) \subset U$. Then there exists $k \in \mathbb{N}$ such that the set $\Lambda_k = \{\lambda \in I \mid i(\lambda) \leqslant k\}$ is uncountable. Let Θ_k^λ be the map of P to $f^{-1}(y_k)$ such that $\Theta_k^\lambda(p) = \Theta^\lambda(p, 1/k)$.

Fix $\nu > 0$ so that $y \notin O(y_k, 2\nu)$ and denote by G the closure of the set $O(y_k, \nu)$. The space of continuous mappings of polyhedron P to the compact $f^{-1}(y_k) \cap \overline{U}$ is separable. Then one can take two mappings $\Theta_k^{\lambda_0}$ and $\Theta_k^{\lambda_1}$ from the uncountable family $\{\Theta_k^\lambda \mid \lambda \in \Lambda_k\}$ so that there exists a homotopy $\varkappa: P \times I \to W \cap f^{-1}(G)$ such that $\varkappa(p, 0) = \Theta_k^{\lambda_0}$ and $\varkappa(p, 1) = \Theta_k^{\lambda_1}$.

Let the map $\gamma: P \times S^1 \to M$ be given by

$$\gamma(p, \alpha) = \begin{cases} \Theta^{\lambda_0}(p, 3\alpha/2\pi k), & \text{if } 0 \leqslant \alpha \leqslant 2\pi/3, \\ \varkappa(p, 3\alpha/2\pi - 1), & \text{if } 2\pi/3 \leqslant \alpha \leqslant 4\pi/3, \\ \Theta^{\lambda_1}(p, 3(1 - \alpha/2\pi)/k), & \text{if } 4\pi/3 \leqslant \alpha \leqslant 2\pi, \end{cases}$$

where S^1 is the unit circle in the plane, parameterized by the polar angle α. Since $\dim P = r+1$, the cycle $z' \in H_{r+1}(P)$ is of infinite order. Then the cycle $z' \otimes 1 \in H_{r+2}(P \times S^1)$ also has infinite order. Let us denote $K = \gamma(P \times S^1)$. Consider the cycle $z \in \check{H}_{r+2}(K; \mathbb{Q})$—the image of $z' \otimes 1$ under the superposition of the coefficient homomorphism $H_{r+2}(P \times S^1) \to H_{r+2}(P \times S^1; \mathbb{Q})$ and the homomorphism γ_*. There exist a point $x_0 \in \gamma(P \times \{0\})$ and a sequence of spheroids $\varphi^i: S^{r+1} \to U \setminus f^{-1}(f(K))$ converging to x_0 and linked with the cycle z. Let us take a number j so large that the spheroid $f \circ \varphi^j: S^{r+1} \to N$ is contractible to a point missing G. Since the set $f(K) \setminus G$ is one-dimensional (it lies in the union $\theta(t_0, I) \cup \theta(t_1, I)$) and hence it is $(r+2)$-negligible in N, we may assume that the contraction of the spheroid $f \circ \varphi^j$ does not touch $f(K)$. Consequently, put $\varphi = \varphi^i$.

Case (5). Fix a point $x' \neq x$ in the connected component of the point x in the fibre $f^{-1}(f(x))$. Take neighborhoods $W \in Ox$ and $W' \in Ox'$ of the points x and x' (as in Case (3)), which are homeomorphic to $\mathbb{R}^m$, and such that $f(W) \subset f(W') \neq N$. Let U be an open neighborhood of x such that $\overline{U} \subset W$ and $U \cong \mathbb{R}^m$.

Suppose that there exist a point $y \in N$ and an essential cycle $z' \in \check{H}_{r+1}(U \cap f^{-1}(y))$. Analogously to Case (4) we may construct a mapping $\gamma: P \times S^1 \to W$, a cycle $z \in \check{H}_{r+2}(K; \mathbb{Q})$ and a sequence of spheroids $\varphi^i: S^{r+1} \to U \setminus f^{-1}(f(K))$, where $K = \gamma(P \times S^1)$, such that every spheroid φ^i is linked with the cycle z and the sequence $\varphi^i(S^{r+1})$ converges to a point $x_0 \in \gamma(P \times \{0\})$.

Take a point $x_0' \in W' \cap f^{-1}(y)$. Since f is locally $(r+1)$-soft, then for sufficiently large i there exist spheroids $\psi^i: S^{r+1} \to M$ such that $\lim_{i \to \infty} \psi^i(S^{r+1}) = x_0'$ and $f \circ \psi^i =$

$f \circ \varphi^i$. Fix a number j such that $\psi^j(S^{r+1}) \subset W'$. Clearly, the cycle $\psi^j_*(S^{r+1})$ does not link with z. Using spheroids ψ^j and φ^j we can construct a spheroid $\varphi: S^{r+1} \to M$ (as in Case (3)) such that the cycle $\varphi_*(S^{r+1})$ is linked with z and the cycle $f \circ \varphi: S^{r+1} \to N$ is homotopically inessential.

Consequently, we have the desired neighborhood U, cycle z and spheroid φ. This completes the proof of Case (5). □

Proof of Theorem 1. For mappings of manifolds k-fiberedness implies homotopical $(k-1)$-regularity [5]. Then Theorem 5(2)–(5) (where $r = k - 1$) imply homotopical ∞-regularity of the mapping f. Since every closed n-regular mapping of complete metric spaces is an n-fibration [4], f is a Serre fibration.

Proof of Theorem 4. Since a k-fibration of manifolds is $(k-1)$-regular [5], Theorem 5(1), (2) and (4) (where $r = k - 1$) imply homotopical ∞-regularity of the mapping f. Since every closed n-regular mapping of complete metric spaces is an n-fibration [4], f is a Serre fibration.

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